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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022

FIRST YEAR [BATCH 2021-24]

Date : 08/03/2022 **PHYSICS (HONOURS)**

Time : 11 am – 1 pm **Paper : I [CC 1]** Full Marks : 50

Answer **any five** questions of the following: [5×10]

1. a) Solve the equation $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$

   b) Solve $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\, logx$ [3+7]

2. a) Find singular points of the differential equation

   $$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + ny = 0$$

   b) Obtain the series solution of the equation $2x(1 - x)\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} + 3y = 0$ [3+7]

3. a) Find the eigen-values and the normalized eigen vectors of the matrix

   $$\begin{bmatrix} 5 & 0 & \sqrt{3} \\ 0 & 3 & 0 \\ \sqrt{3} & 0 & 3 \end{bmatrix}$$

   b) Prove that any two eigen vectors of a real symmetric matrix are orthogonal provided the corresponding eigenvalues are different. [7+3]

4. a) Solve the coupled differential equations:

   $$\frac{dx}{dt} + 2x - 3y = 5t\ ;\ \frac{dy}{dt} - 3x + 2y = 2e^{2t}$$

   b) Verify Cayley-Hamilton Theorem for the following matrix:

   A=$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and use the theorem to find A^{-1}. [6+4]

5. a) Find div(F) and Curl(F), where $F = xy^2\hat{\imath} + xz^2\hat{\jmath} + 6y^2z\hat{k}$ at (2, 0, -1).

   b) Prove $\vec{A}.(\vec{A} \times \vec{B}) = 0$

   c) A vector field is given by $\vec{F} = \sin y\, \hat{\imath} + x(1 + \cos y)\, \hat{\jmath}$; evaluate $\int_c F\,.dr$ over a path given by $x^2 + y^2 = a^2$ and $z = 0$. [4+2+4]

6. a) Evaluate $\iint \vec{F}\,.\,d\bar{s}$; where $F = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and s is the part of plane $6x + 2y + 3z = 12$.

b) If $u = xyz, v = x^2 + y^2 + z^2$ and $w = x + y + z$; find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ [5+5]

7. a) Test $V_3(R)$ linearly dependent or independent, where $\propto_1 = (1, -1,1), \propto_2 = (1,0,0)$,

$\propto_3 = (0,1,0), \propto_4 = (0,0,1), \in V_3(R)$

b) If inner product define by $< p, q > = \int_{-1}^{1} p(x)q(x)dx$, where

$p(x) = x$,

$q(x) = x^2$; Show P and q are orthogonal. [5+5]

8. a) Derive the relation between unit vectors of Cartesian and spherical polar coordinate.

b) Evaluate ∇ in spherical polar coordinate. [5+5]

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